

Hunting for a scalar glueball in exclusive B decays

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Abstract. Although glueballs, as one of the type of exotic hadrons allowed by QCD, have been well established on the lattice, experimental searches up to this date for bound states of gluons have only produced controversial signals. In this work, using flavor SU(3) symmetry for the light quarks validated by the available experimental data, we propose an intuitive way to hunt for a scalar glueball in exclusive B decays. In the presence of mixing between the glueball and ordinary scalar mesons, we point out that the scalar glueball content amounts to the ratio of B -decay branching fractions. We discuss the implication from the recently available experimental data and show the sensitivities of B decays as a probe to the scalar structures. The on-going LHCb experiment and forthcoming Super KEKB factory would allow access to precisely establishing the mixing pattern among the scalars, and may allow one to disentangle the long-standing puzzle concerning the existence and mixings of the scalar glueball predicted by QCD.

1 Introduction

QCD allows the existence of hadronic states beyond the quark model description of mesons and baryons. Among these exotic states, glueballs are bound states made of the color force carriers only. Lattice QCD simulations have suggested the mass of the lowest-lying scalar glueball around 1.5–1.8 GeV [1–9]. Despite several possible candidates in this mass region, the existence of a scalar glueball state is still under debate, largely because of the fact that the lowest-lying scalar glueball has the same quantum numbers as the QCD vacuum, and thereby mixes with ordinary quark-antiquark states.

Most glueball studies available in the literature have focused on decay properties and the production in low-energy processes. In fact, the study of the production in B decays is another powerful way to uncover the mysterious structure of scalar mesons and figure out the gluon component inside [10–15].

The motivation of this work is to provide an up-to-date analysis of B decays into a scalar meson plus a J/ψ , particularly in the light of the recent data on $B/B_s \rightarrow J/\psi \pi^+ \pi^- / K^+ K^-$ decays from the LHCb, Belle and D0 Collaborations [16–20]. In view of these, we will suggest an intuitive way for the identification of a glueball.

In the following, we shall consider three scalar mesons $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ all having potentially a large glue content, see, for instance, refs. [21–23]. These three mesons, together with the isotriplet $a_0(1450)$ and isodoublet $K_0^*(1430)$ can form an SU(3) nonet made of $\bar{q}q$, with one additional state arising from the mixing with the glueball [24]. From this viewpoint, the isosinglet scalar meson among $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ is expanded as

$$|f_0\rangle = \alpha_1|G\rangle + \alpha_2|\bar{s}s\rangle + \alpha_3|\bar{n}n\rangle, \quad (1)$$

in which the coefficient α_1 is the measure of the glue content. Here, n denotes the light quark flavors up and down and we work in the isospin limit in what follows. When other Fock states are included, it is likely that $\alpha_1^2 + \alpha_2^2 + \alpha_3^2 < 1$ due to the unitarity condition. However since most of the available Lattice QCD simulations suggest that the mass of the scalar glueball is in the range from 1.5 GeV to 1.8 GeV, it is reasonable to believe that glueball will dominate in one of the mesons discussed here. In this case, it is plausible to neglect the subdominant components, leading to $\alpha_1^2 + \alpha_2^2 + \alpha_3^2 \simeq 1$. Other alternative mixing schemes of scalar mesons are also proposed in the literature, and, for instance, ref. [25] has discussed the route in which the scalar glueball may mix with the $f_0(980)$. However our following discussion is still valid as long as the glue content dominates in one of the scalar mesons.

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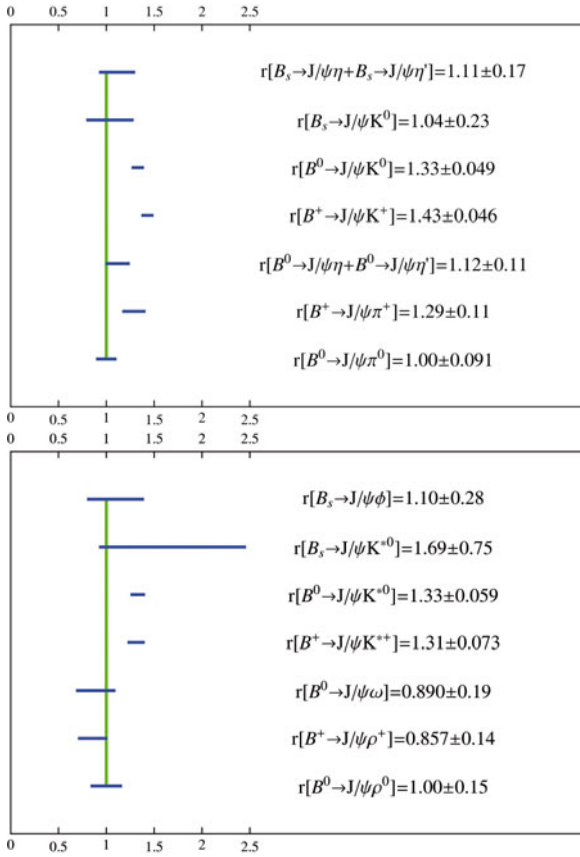


Fig. 1. Ratios of branching fractions of $B \rightarrow J/\psi P$ and $B \rightarrow J/\psi V$. The vertical lines denotes unity and correspond to the SU(3) symmetry limit.

2 General analysis based on SU(3) symmetry

To start, we will assume flavor SU(3) symmetry for the light u, d, s quarks in the $B \rightarrow J/\psi M$ decay amplitudes. This symmetry has been partly tested in a few $B \rightarrow J/\psi P$ and $B \rightarrow J/\psi V$ processes [26, 27] and a good agreement with the data is found. The underlying nature of these processes at the quark level is governed by the $b \rightarrow \bar{c}cs$ or $b \rightarrow \bar{c}cd$ transitions whose matrix elements can be related using the flavor symmetry. For a better comparison to be made in the following, we define the ratio

$$r[B_q \rightarrow J/\psi P_{q\bar{q}'}] = \frac{|V_{cd}|^2 |C_{\pi^0}|^2 \tau(B^0)}{|V_{cq'}|^2 |C_P|^2 \tau(B_q)} \times \frac{\mathcal{B}(B_q \rightarrow J/\psi P_{q\bar{q}'})}{\mathcal{B}(B^0 \rightarrow J/\psi \pi^0)}, \quad (2)$$

whose deviation from unity directly arises from the SU(3) symmetry breaking effects. In this ratio, $C_P = 1$ (except for $-C_{\pi^0} = C_{\eta_{q\bar{q}}} = 1/\sqrt{2}$) is the flavor wave function factor. The ratios for the $B \rightarrow J/\psi V$ processes are defined in a similar way. Using the relevant experimental data for the branching fractions of the various channels [28–30], we collect the results for these ratios in fig. 1. The vertical lines in this figure denote unity and thus corresponds to the exact SU(3) symmetry limit.

Three observations can be made from the results presented in fig. 1. Firstly, the current uncertainties in the B_s decays are large but may get significantly reduced due to the large amount of data accumulated from the LHC and future Super B factories. Secondly, the SU(3) symmetry holds well in the $b \rightarrow d$ processes (namely $B \rightarrow (J/\psi \pi, J/\psi \eta^{(\prime)})$ and $B_s \rightarrow J/\psi K$), and as well as in the $b \rightarrow s$ transition ($B \rightarrow J/\psi K$ and $B_s \rightarrow J/\psi \eta^{(\prime)}$). Last, the excess of the branching ratios for the $b \rightarrow s$ processes, roughly 30%, is the same in both $B \rightarrow J/\psi P$ and $B \rightarrow J/\psi V$ decays.

After validating the flavor SU(3) symmetry, we now explore the consequences in the application to $B \rightarrow J/\psi S$ decays. Suppose in the near future we are equipped with the following experimental data:

$$\begin{aligned} \mathcal{B}_1 &= \mathcal{B}(B^0 \rightarrow J/\psi K_0^*(1430)), \\ \mathcal{B}_2 &= \mathcal{B}(B_s^0 \rightarrow J/\psi K_0^*(1430)), \\ \mathcal{B}_3 &= \mathcal{B}(B_s^0 \rightarrow J/\psi f_0), \\ \mathcal{B}_4 &= \mathcal{B}(B^0 \rightarrow J/\psi f_0), \end{aligned} \quad (3)$$

where the second quantity can also be replaced by $\mathcal{B}(B^- \rightarrow J/\psi a_0^-(1450))$. The first and third processes are induced by the $b \rightarrow s$ transition, while the other two arise from the $b \rightarrow d$ transition. In the SU(3) symmetry (to be more specific the U-spin symmetry which interchanges the s and d quarks) limit, $\mathcal{B}_1 = \mathcal{B}_2$, but in order to account for the symmetry breaking effects that can reach 30% as we have shown, we will retain the differences in \mathcal{B}_1 and \mathcal{B}_2 . This treatment will refine our analysis based on the SU(3) symmetry and greatly reduce the systematic errors in the analysis. Using these four branching ratios, we also introduce the quantity

$$\mathcal{R} = 1 - \mathcal{B}_3/\mathcal{B}_1 - 2\mathcal{B}_4/\mathcal{B}_2. \quad (4)$$

In the leading-Fock-state approximation, a scalar glueball is composed of two constituent gluons. In exclusive B decays, the two gluons can be emitted from either the heavy b quark or the light quark. The emission of a collinear gluon from the heavy b quark is suppressed by $1/m_b^2$. Since the initial state does not contain any valence gluon, in order to generate the glueball the quarks have to be annihilated via the QCD interaction. Compared to the form factor of B to an ordinary $\bar{q}q$ transition, such a contribution is suppressed by $\alpha_s(m_b \Lambda_{QCD})$, where the scale in α_s has been set to the typical scale in the transition. The calculation in the perturbative QCD approach shows that the B -to-glueball form factor is suppressed by a factor of 6–10 [31, 32].

In the following discussion, we will neglect the small contributions from the glueball content, and thus only the $\bar{n}n$ ($\bar{s}s$) component will contribute in B (B_s) decays into a scalar meson plus a J/ψ . As an important consequence, $\mathcal{B}_3/\mathcal{B}_1 = \alpha_s^2$ and $2\mathcal{B}_4/\mathcal{B}_2 = \alpha_s^2$, while \mathcal{R} directly reflects the size of the glueball component. *The significant deviation of \mathcal{R} from 0 may be a signal for a glueball.*

3 Implications from the present data

In ref. [16], the Belle Collaboration reported the observation of a scalar mesonic state f_X from the process $B_s \rightarrow J/\psi f_X \rightarrow J/\psi \pi^+ \pi^-$ with a significance of 4.2σ ,

$$\mathcal{B}(B_s \rightarrow J/\psi f_X \rightarrow J/\psi \pi^+ \pi^-) = (0.34_{-0.15}^{+0.14}) \times 10^{-4}. \quad (5)$$

The mass and width of this resonance are determined as

$$\begin{aligned} m_{f_X} &= (1.405 \pm 0.015_{-0.007}^{+0.001}) \text{ GeV}, \\ \Gamma_{f_X} &= (0.054 \pm 0.033_{-0.003}^{+0.014}) \text{ GeV}. \end{aligned} \quad (6)$$

Subsequently, the LHCb Collaboration has found a similar resonance,

$$\begin{aligned} m_{f_X} &= (1.4751 \pm 0.0063) \text{ GeV}, \\ \Gamma_{f_X} &= (0.113 \pm 0.011) \text{ GeV}. \end{aligned} \quad (7)$$

The branching ratio (BR) of $B_s \rightarrow J/\psi f_X$ is roughly 4% of the BR for $B_s \rightarrow J/\psi \phi$. Remembering that $\mathcal{B}(B_s \rightarrow J/\psi \phi) = (1.09_{-0.23}^{+0.28}) \times 10^{-3}$ [28], we note that the LHCb result is consistent with the Belle measurement.

The measured branching ratio of $B_s \rightarrow J/\psi f_X \rightarrow J/\psi \pi^+ \pi^-$ is helpful in determining the mixing coefficient α_2 in f_X together with

$$\mathcal{B}(B^0 \rightarrow J/\psi K^0) = (8.71 \pm 0.32) \times 10^{-4}. \quad (8)$$

Under the assumption of factorization, we extract the coefficient α_2 as

$$\begin{aligned} \alpha_2 &= (0.27 \pm 0.13) \times \frac{F_1^{B \rightarrow K}(m_{J/\psi}^2)}{0.53} \\ &\times \frac{1.22}{F_1^{B_s \rightarrow f_0(\bar{s}s)}(m_{J/\psi}^2)} \times \sqrt{\frac{10\%}{\mathcal{B}(f_X \rightarrow \pi^+ \pi^-)}}, \end{aligned} \quad (9)$$

where we have used the calculation of form factors from refs. [33,32]: $F_1^{B \rightarrow K}(m_{J/\psi}^2) = 0.53$ and $F_1^{B_s \rightarrow f_0(\bar{s}s)}(m_{J/\psi}^2) = 1.22$, while variations of the form factors will change the extracted coefficient. The uncertainties shown in the parenthesis are from the nonfactorizable contributions and have been conservatively taken as large as 50%. The decay branching fraction of $f_X \rightarrow \pi^+ \pi^-$ is an important input in the analysis and we have used 10% for illustration.

4 Comparison with theory

For illustration, we will consider two widely discussed mixing mechanisms of the scalar mesons and for an overview of alternative schemes see refs. [24,34–36] and many references therein. Because the decay width of the $f_0(1500)$ is not compatible with the ordinary $\bar{q}q$ state, it is claimed that $f_0(1500)$ is primarily a scalar glueball [21,22], and

the mixing matrix through fitting the data of two-body decays of scalar mesons is

$$\begin{pmatrix} f_0(1710) \\ f_0(1500) \\ f_0(1370) \end{pmatrix} = \begin{pmatrix} 0.36 & 0.93 & 0.09 \\ -0.84 & 0.35 & -0.41 \\ 0.40 & -0.07 & -0.91 \end{pmatrix} \begin{pmatrix} G \\ \bar{s}s \\ \bar{n}n \end{pmatrix}. \quad (10)$$

Based on the SU(3) assumption for scalar mesons and the quenched Lattice QCD results [9], Cheng *et al.* [23] re-analyzed all existing experimental data and derived the mixing coefficient matrix as

$$\begin{pmatrix} f_0(1710) \\ f_0(1500) \\ f_0(1370) \end{pmatrix} = \begin{pmatrix} 0.93 & 0.17 & 0.32 \\ -0.03 & 0.84 & -0.54 \\ -0.36 & 0.52 & 0.78 \end{pmatrix} \begin{pmatrix} G \\ \bar{s}s \\ \bar{n}n \end{pmatrix}. \quad (11)$$

Here, the $f_0(1710)$ tends to be a glueball. This is very different from the first matrix of mixing coefficients in eq. (10), while both schemes can well explain the data on the production in J/ψ and decays of the f_0 .

The mixing scheme of eq. (11) predicts a much larger production branching ratio for $J/\psi \rightarrow \gamma f_0(1710)$ than $J/\psi \rightarrow \gamma f_0(1500)$ and implies a relatively pure glueball around 1.7 GeV. In contrast, the mixing scheme of eq. (10) suggests that those two nearby states $f_0(1500)$ and $f_0(1710)$ both have sizeable glueball components. It can be understood that the interferences between the glueball and $\bar{q}q$ production would lead to an enhanced production rate for $J/\psi \rightarrow \gamma f_0(1710)$, but a suppression of $J/\psi \rightarrow \gamma f_0(1500)$. Such an ambiguity reflects the lack of knowledge on the glueball- $\bar{q}q$ coupling in the scalar sector. Qualitatively speaking, it is strongly model-dependent to determine the glueball component of scalar mesons in their strong productions and strong decays. In this sense, it is interesting to recognize the advantages of probing the flavor components of the scalar mesons in B decays. To be more specific, in the decay of $B_s \rightarrow J/\psi f_0$ the mixing coefficients in the second column, α_2 's defined in eq. (1), will be projected out by the weak transitions. Thus, they provide a natural filter of the $\bar{s}s$ component.

Apart from the measurements on $B_s \rightarrow J/\psi \pi^+ \pi^-$, both the LHCb and D0 Collaborations have measured the branching ratio of the process $B_s \rightarrow J/\psi K^+ K^-$ [18,19], in which no significant evidence for any scalar resonance decaying into $K^+ K^-$ is found. Thus, it may be hard to interpret the $f_0(1710)$ as an $\bar{s}s$ state since in this case the $f_0(1710)$ mainly decays into $K^+ K^-$. From this viewpoint, it seems that the mixing in eq. (10) is less favored compared to the one in eq. (11), where the production of the glueball dominated $f_0(1710)$ is expected to be suppressed.

The present experimental precision does not allow a conclusion for the $f_0(1500)$. Although in the mixing scheme of eq. (11), the $f_0(1500)$ is favored to be produced via its $\bar{s}s$ component, its decay branching ratio to $K^+ K^-$ is relatively small, *i.e.* $(8.6 \pm 1.0)\%$ [28]. With higher statistics available in the future, a determination of the relative production rates for the $f_0(1710)$ and the $f_0(1500)$ in $B_s \rightarrow J/\psi \pi^+ \pi^-$ should be able to provide crucial information about their internal structure.

Supposing that the absence of $f_0(1710)$ in K^+K^- is indeed due to the dominance of an internal glueball component, one notices that the such a scenario is consistent with the recent Lattice QCD calculation of ref. [37], where the $f_0(1710)$ as a glueball candidate would have a large production rate in J/ψ radiative decays, *i.e.* $J/\psi \rightarrow \gamma f_0(1710)$. It is also in support from the coupled channel study of the S-waves meson-meson scattering [38] (for different opinions see refs. [39,40]), in which the $f_0(1710)$ and a pole at 1.6 GeV, which is an important contribution to the $f_0(1500)$, are identified as the scalar glueballs.

Regarding the f_X resonance discovered by Belle and LHCb, we explore two cases since the masses and widths of both $f_0(1370)$ and $f_0(1500)$ are close to the experimental values:

- i) The f_X can be the $f_0(1500)$. From the PDG tables [28], we quote

$$\mathcal{B}(f_0(1500) \rightarrow \pi^+\pi^-) = \frac{2}{3} \times 34.8\% = 23.2\%, \quad (12)$$

which implies

$$|\alpha_2| = (0.18 \pm 0.09). \quad (13)$$

Such a small value seems to favor the mixing matrix shown in eq. (10).

- ii) The f_X can be the $f_0(1370)$. The PDG quote that the $f_0(1370) \rightarrow \rho\rho$ is its main decay mode. Both WA102 [41] and BES-II [42] found that the branching ratio fraction of $f_0(1370) \rightarrow \pi^+\pi^-$ over $f_0(1370) \rightarrow K^+K^-$ is small, *i.e.* $\sim 20\%$ [42]. If this is the case, the extracted coefficient α_2 is of a similar size as the value in eq. (13). In such a situation, it seems hard to distinguish the mixings given in eqs. (10) and (11).

5 Future improvements

Although the present experimental status does not allow us to make a definite conclusion on the f_X , we would expect that the situation will be greatly improved in the future. As we have shown above, the measurement of branching ratios of the $B_s \rightarrow J/\psi f_0$ with high statistics will be able to pin down the flavor components of the scalars. Therefore, a precise measurement of the relative production rates of all (or some of) those scalars in $B_s \rightarrow J/\psi f_0$ will be an ideal $\bar{s}s$ filter for the determination of the $\bar{s}s$ components inside those scalar mesons. It is also possible to use the $B \rightarrow J/\psi f_0$ decays as a flavor filter for the non-strange $\bar{q}q$ components similar to that in $B_s \rightarrow J/\psi f_0$. A combined measurements of $B_s \rightarrow J/\psi f_0$ and $B \rightarrow J/\psi f_0$ will be very selective to scalar mixing models and can be compared with the scalar production mechanisms studied in, *e.g.*, $J/\psi \rightarrow \gamma f_0$.

Generically the branching fractions of the $b \rightarrow \bar{c}cd$ processes are suppressed by $V_{cd}^2/V_{cs}^2 \sim 0.04$, which we suppose would be compensated by the large luminosity of the future experiments. Based on the 1 fb^{-1} of data sample collected at 7 TeV, the LHCb Collaboration has analyzed the resonant components in $B \rightarrow J/\psi \pi^+\pi^-$ [20]. The Super KEKB factory is expected to gather about 50 ab^{-1} of data, which is two orders of magnitude larger than the data sample collected on the KEKB collider [43]. With such a high statistic data base, one might gain access to $B_s/B_d \rightarrow J/\psi f_0(\gamma\gamma)$ in which the scalar meson is reconstructed in the two-photon final state. Compared to the $B_s/B \rightarrow J/\psi f_0(\pi^+\pi^-, K^+K^-)$, in which the decay of the f_0 is not under control due to the unknown contributions from the glueball, the $B_s/B_d \rightarrow J/\psi f_0(\gamma\gamma)$ is cleaner. Due to the fact that the gluons are free of electromagnetic interaction, the glueball component will not contribute. Thus, the decay matrix elements of the three f_0 states can be determined by the mixing coefficients and electric charges of the flavor components.

6 Conclusion

To summarize, using the available experimental data we have demonstrated that the flavor SU(3) symmetry for the light quarks holds quite well and can be applied to the study of B decays into a scalar meson plus a charmonium. Our analysis suggests that such a process would serve as an ideal flavor filter for probing the quark contents of scalar mesons. In the presence of mixings between a glueball and ordinary $\bar{q}q$ mesons, we show that the mixing parameters can be extracted and explicitly related to experimental data from the LHCb, Belle and D0 Collaborations. Although the present experimental data sample does not allow a solid conclusion on all those states, we have shown the sensitivities of such a probe to the scalar structures. The future Super KEKB factory would allow access to establishing the mixing pattern among those three scalars, and possibly allow the long-standing puzzle concerning the existence and mixings of the scalar glueball predicted by QCD to be disentangled.

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